On Dense Phase Voidage and Bubble Size in High Pressure Fluidized Beds of Fine Powders

Dense phase voidage, ϵ_D, dense phase superficial gas velocity, u_{Do}, and absolute bubble rise velocity, u_B, were measured at pressures up to 8300 kPa in a pilot-scale, fluidized bed of Group A and boundary Group A/B powders. The mean equivalent bubble diameter, d_b, near the top of the bed was inferred from u_B and known bed operating conditions. Increased pressure at fixed superficial gas velocity, u_o, increased ϵ_D and u_{Do} and decreased d_b for Group A powders. The marked decrease in inferred maximum bubble size, $d_{b\,max}$, with increased pressure could not be explained by a decrease in gas contributing to bubble flow, u_{Bo}, but rather appeared to be the result of a bubble instability phenomenon limiting bubble growth.

A. W. WEIMER and G. J. QUARDERER

Michigan Applied Science and Technology Labs
Dow Chemical U.S.A.
Midland, MI 48640

SCOPE

A complete understanding of pressurized fluidized beds requires knowledge of the effect of pressure on fundamental fluidization hydrodynamic properties. It is well known that increased pressure increases ϵ_D and u_{Do} and decreases d_b in fluidized beds of Group A powders. Of fundamental importance is the extent to which these pressure effects occur and the reasons for them.

Considerable interest has been given to the existence of a maximum stable bubble diameter, $d_{b_{max}}$, in pressurized systems resulting from bubble splitting due either to particle "pickup" (Harrison et al., 1961) from the lower surface of rising bubbles or to Taylor instability (Clift et al., 1974) of the roof of the bubble. Bubbles have been seen to split both from above and from below in fluidized beds with the dominant mechanism of splitting being disputed. An alternate explanation for smaller bubbles is that bubble growth is limited by a reduced amount of gas contributing to bubble flow. Whether the reduced bubble size accompanying increased pressure results from reduced u_{Bo} and/or instabilities limiting bubble growth cannot be determined without simultaneous measurement of dense and bubble phase properties. To date, such measurements have not been made at pressures above 1200 kPa.

In this work, ϵ_D, u_{Do}, and $d_{b\,max}$ were measured in a pilot-scale fluidized bed of 66 μm, 108 μm, and 171 μm powders at pressures up to 8300 kPa. Dense phase voidage, ϵ_D, and u_{Do} were determined at all pressures by the bed collapse technique (Rietema, 1967), with the collapsing bed monitored by a high frequency response nuclear radiation density gauge. The absolute rise velocity of bubbles, u_B, near the top of the bed was determined from cross-correlation analysis of multiple differential pressure, ΔP, transducer outputs. The bubble diameter $d_{b\,max}$ was then inferred from u_B and known bed operating conditions. The experimental ϵ_D and u_{Do} were compared to estimated values from available correlations, which account for density effects. In addition, a detailed analysis of the experimental results was done to determine if particle "pickup," Taylor instabilities, or reduced u_{Bo} could explain the observed decrease in $d_{b\,max}$ accompanying increased pressure.

CONCLUSIONS AND SIGNIFICANCE

The magnitude of the pressure effect on ϵ_D, u_{Do}, and $d_{b\,max}$ was strongly dependent on the mean particle size, d_p. There was a substantial pressure effect for the 66 μm powder, a modest pressure effect for the 108 μm powder, and essentially no pressure effect for the 171 μm powder at pressures up to 8,300 kPa. For the 66 μm powder, ϵ_D increased from 0.53 at 100 kPa to 0.74 at 6,200 kPa. This increase in ϵ_D was accompanied by a fivefold increase in u_{Do} to $u_{Do} > 7u_{mf}$ at 6,200 kPa. Over this same pressure range, $d_{b\,max}$ decreased from 0.058 m to less than 0.01 m. For the 108 μm powder, a voidage increase from $\epsilon_D = 0.44$ at 100 kPa to $\epsilon_D = 0.51$ at 8,300 kPa was accompanied by an 11% increase in u_{Do}. The decrease in $d_{b\,max}$ with increased pressure was more gradual than that for the 66 μm powder.

Experimentally obtained parameters were compared to those predicted by available correlations and an analysis was done to determine if particle "pickup," Taylor instabilities, or reduced u_{Bo} could explain the observed decrease in $d_{b\,max}$, with increased pressure. The relationship between ϵ_D and u_{Do} for all powders at all pressures was reasonably well estimated from the correlations of Abrahamsen and Geldart (1980b) and Kmiec (1982). It was concluded that the measured increase in u_{Do} (decrease in u_{Bo}) with increased pressure for the 66 and 108 μm powders could not account for the marked decrease in $d_{b\,max}$. Instead, a limited $d_{b\,max}$ existed and decreased with increased pressure. Predicted $d_{b\,max}$ from the particle "pickup" theory (Harrison et al., 1961) underestimated measured values by a large degree. Although sufficient data were not available to predict $d_{b\,max}$ from the Taylor instability theory (Clift et al., 1974), measured changes in ϵ_D were approximate to the ϵ_D changes required to limit $d_{b\,max}$ at increased pressures.

Correspondence concerning this paper should be addressed to A. W. Weimer.

BACKGROUND

Interest in fluidized-bed behavior of fine powders [Group A according to Geldart's (1972) classification] at elevated pressures has increased substantially in recent years. A number of researchers (Abrahamsen and Geldart, 1980a, b, c; Dry et al., 1983; Geldart and Abrahamsen, 1980; Rowe and Yacono, 1976; Rowe et al., 1978; and Simone and Harriott, 1980) have found the fluidization characteristics of fine powders to deviate significantly from behavior predicted by the two-phase theory (Toomey and Johnstone, 1952). Dense phase voidage, ϵ_D, is greater and bubble diameter, d_b, smaller in fluidized beds of Group A powders relative to fluidized beds of Group B powders at equivalent excess superficial velocities (i.e., equivalent $u_o - u_{mf}$). These deviations increase with pressure.

Crowther and Whitehead (1978), Guedes de Carvalho et al. (1978), Sobreiro and Monteiro (1982), and King and Harrison (1982) investigated the influence of pressure on minimum fluidization and minimum bubbling variables for Group A and Group B powders at pressure to 6,900 kPa. For Group A powders, minimum fluidization velocity, u_{mf}, and voidage, ϵ_{mf}, were insensitive to pressure, whereas minimum bubbling velocity, u_{mb}, and voidage, ϵ_{mb}, increased with increased pressure. For Group B powders, u_{mb} and u_{mf} were identical and decreased with increased pressure, while ϵ_{mb} and ϵ_{mf} were identical and unaffected by pressure.

Although bed properties at minimum bubbling conditions approximate dense phase properties in a bubbling fluidized bed, dense phase properties are more meaningful. Subzwari et al. (1978) investigated ϵ_D in a pressurized, two-dimensional, fluidized bed of Group A and border Group A/B powders. For the 60 μm Group A powder, ϵ_D increased 11% as the operating pressure was increased from 100 to 700 kPa. Pressure had little effect on the 100 μm Group A/B powder. Guedes de Carvalho (1981) found that ϵ_D in a bed of 48 μm Group A powder increased 15% as the bed pressure was increased from 100 to 2,200 kPa. Barreto et al. (1983) found ϵ_D to increase 20% for a 40 μm zeolite powder as bed pressure increased from 100 to 2,000 kPa.

It is generally observed that with increasing pressure, as ϵ_D and u_{Do} increase, bubbles are smaller in size. This is due either to reduced bubble flow (i.e., reduced $u_o - u_{Do}$) and/or to an instability phenomenon that limits bubble growth. Considerable interest has been given to the existence of a maximum stable bubble size in pressurized Group A systems resulting from bubble splitting due either to particle "pickup" (or saltation) (Harrison et al., 1961; Matsen, 1973) from the lower surfaces of rising bubbles or an increased Taylor instability (Clift et al., 1974, Clift and Grace, 1972; Rice and Wilhelm, 1958) of the roof of the bubbles. Bubbles have been seen to split both from above and from below in fluidized beds with the dominant mechanism of splitting being disputed.

The size of bubbles in pressurized (100 to 700 kPa) fluidized beds has been measured by Subzwari et al. (1978). For beds of 60 μm Group A powder, the size and number of bubbles decreased substantially with increased pressure. The maximum observed bubble size decreased 37% over the pressure range 100 to 700 kPa, and there was an accompanying 29% decrease in bubble flow. A maximum stable bubble size was observed with bubbles seen, in all cases, to split from the roof. Recently, Barreto et al. (1983) utilized x-ray absorption techniques to measure mean equivalent bubble diameters at pressures to 1,200 kPa for six Group A powders, ranging in size from 40 to 98 μm. Reductions in mean equivalent bubble diameter approached 50% for the 98 μm powder as pressure was increased from 100 to 1,200 kPa. Although bubble size was not measured, Varadi and Grace (1978), through qualitative observations, showed that bubble size was unaffected by pressure and that a maximum stable bubble size did not exist for Group B sand particles (250–295 μm) at pressures up to 2,200 kPa. Some splitting of bubbles did occur, but there was no apparent increase in the incidence of splitting at the higher bed pressures. When splitting did occur, it did so from the roof. The observation by both Subzwari et al. (1978) and Varadi and Grace (1978) that pressure had a much greater effect on bubbles in systems of Group A powders than Group B powders was confirmed by King and Harrison (1980). Their observation of bubbles splitting from the roof supports the Taylor instability theory of bubbles splitting from above. On the other hand, Barreto et al. (1983) found the dominant mode of bubble splitting in Group A powders to be from below. Their experimental observations are consistent with the particle "pickup" bubble instability theory, which envisages particles swept up into bubbles from below, causing bubbles to split at the base.

Generally, bubbles in fluidized beds of Group A powders are smaller at higher pressure, but Group B powders are unaffected by pressure, at least up to 3,000 kPa. Since detailed studies of boundary Group A/B and Group B powders at pressures above 3,000 kPa have not been reported, the pressure effect–particle size relationship has not been completely determined. Furthermore, whether the reduced bubble size accompanying increased pressure results from reduced bubble flow and/or instabilities limiting bubble growth, or a combination thereof, has not been quantified.

Most pressurized experimental work reported to date has been qualitative with quantitative measurements of d_b and ϵ_D limited to pressures not exceeding 2,500 kPa. Simultaneous measurements of ϵ_D, u_{Do}, and d_b have not been made at pressures above 1,200 kPa. Without simultaneous measurements of dense and bubble phase properties at higher pressures, the effect of pressure on fluidized systems cannot be completely understood.

The work reported here includes measurements of dense and bubble phase properties with Group A and Group A/B boundary powders at pressures up to 8,300 kPa in a pilot-scale fluidized bed. Quantities measured include ϵ_D, u_{Do}, and u_B, with d_b inferred from u_B. Experimentally obtained parameters are compared to those predicted by available correlations, and an analysis is done to determine if particle "pickup," Taylor instabilities, reduced bubble flow, or a combination thereof can explain the observed decrease in bubble size with increased pressure.

EXPERIMENT

Apparatus

The experiments were carried out in a steel fluidized bed of height 2.7 m and dia. 0.13 m charged with enough powder so the expanded bed height during fluidization did not exceed 2.4 m at the highest superficial gas velocities. Feed gas was fed to the bed through a compressor allowing operation to 8,500 kPa. The distributor was porous steel, ensuring even gas distribution and the formation of small bubbles at the inlet. Pressure in the bed was maintained by a back-pressure control valve located downstream of the bed. Fast action on-off solenoid valves were located upstream and downstream of the pressurized bed and could isolate it instantaneously. A schematic of the experimental system is shown in Figure 1.

Physical Properties of Powder and Gas

Properties of the fluidized granular powders are given in Tables 1 and

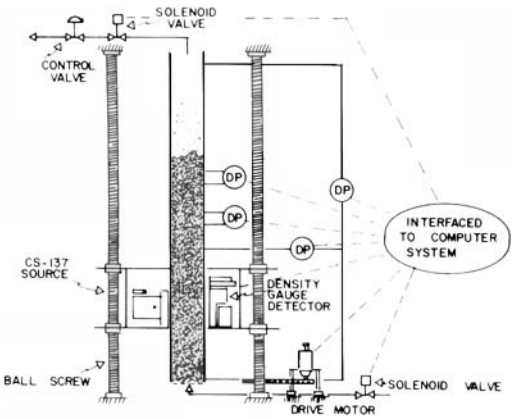

Figure 1. Schematic of experimental apparatus.

TABLE 1. PARTICLE DENSITY AND SIZE DISTRIBUTIONS

Particle Density, ρ_p: 850 kg/m³

Particle Size, $\bar{d}_p$:	66 μm wt. %	66 μm Thru	66 μm On	108 μm wt. %	108 μm Thru	108 μm On	171 μm wt. %	171 μm Thru	171 μm On
Distributions:	79.3	75 μm	63	43.3	150 μm	125	0.1	250 μm	210
	17.5	63	53	25.3	125	106	48.6	125	106
	3.2	53	45	11.3	106	90	35.5	180	150
	100.0								
				7.1	90	75	11.2	150	125
				13.0	75	63	4.6	125	106
				100.0			100.0		

TABLE 2. MINIMUM FLUIDIZATION PROPERTIES OF POWDERS

Mean Particle Size, $\bar{d}_p$: Pres. (kPa)	66 μm u_{mf}(m/s × 10²)	66 μm ϵ_{mf}	108 μm u_{mf}(m/s × 10²)	108 μm ϵ_{mf}	171 μm u_{mf}(m/s × 10²)	171 μm ϵ_{mf}
100	0.255	0.512	0.706	0.421	1.28	0.404
2,100	0.249	0.525	0.642	0.435	1.23	0.407
4,100	0.241	0.530	0.532	0.431	1.09	0.409
6,200	0.233	0.528	0.501	0.437	0.988	0.409
8,300	0.228	...	0.498	0.446	...	...

2. The 66 μm powder was well within the Group A classification, while the 108 and 171 μm were borderline Group A/B powders with the former closer to Group A and latter closer to Group B. The measurement of minimum fluidization velocity was done at all pressures by the standard pressure drop technique (Richardson, 1971), starting with a well-fluidized bed and gradually reducing gas flow. The fluidizing medium was a gas mixture having an average molecular weight of $M_w = 17$ and a viscosity of $\mu_g = 1.56 \times 10^{-5}$ Pa · s. All experiments were performed at ambient temperature.

Instrumentation

Four rapid response DP transducers (Validyne, model DP15) were strategically located on the fluidized bed so as to measure critical hydrodynamic parameters (see Figure 2). The fast response (<1 ms) was desirable for DP3 and DP4; however, slower response times would have sufficed for the standard differential pressure measurements obtained from DP1 and DP2. The main purpose of DP3 and DP4, as will be discussed later in this paper, was to determine the absolute rise velocity and hence the size of bubbles. For this purpose it was necessary that they both be located in regions below the top of the fluidized bed. In practice, the exact location, distance between transducers (distance L, shown in Figure 2) and measuring interval (distance l, shown in Figure 2) are somewhat arbitrary and depend on the height and diameter of the fluidized bed being studied. Here DP3 and DP4 had measuring intervals of $l = 0.152$ m and were separated by a distance of $L = 0.610$ m. DP4 was located so that its upper tap was always just below the lowest anticipated expanded bed height, L_f.

To prevent solids from plugging the pressure taps, filter fittings (Cajon Gauge Protector Snubber Adapters, Series SA) were located in the pressure taps as close as possible to the wall of the fluidized bed. A rapid-response (1 ms time constant) nuclear density gauge (Ohmart Densart 3400, 500 mCi Cs-137 point source) was mounted to a movable assembly mechanism constructed so that the gauge could range over the entire height of the fluidized bed. The application of this type gauge is described in detail elsewhere (Weimer, 1980). An on-line microprocessor was utilized to monitor (up to 100 points/s) and to perform statistical calculations on the rapidly fluctuating signals from transducers DP3 and DP4 and the nuclear density gauge.

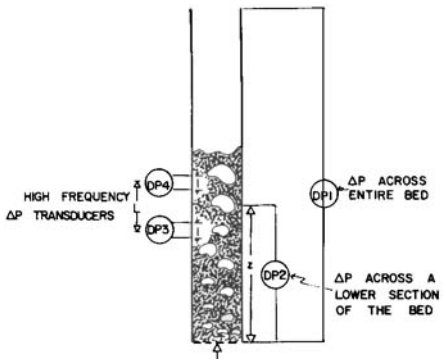

Figure 2. Location of differential pressure measurements on a fluidized bed.

EXPERIMENTAL MEASUREMENTS AND ANALYSIS OF RESULTS

Dense Phase Voidage, ϵ_D, and Superficial Gas Velocity, u_{Do}

The dense phase voidage, ϵ_D, was determined at all pressures by the bed collapse technique (Rietema, 1967). The bed was initially well fluidized with gas flow sufficient to maintain vigorous bubbling; at a given instant ($t = 0$) the on-line microprocessor simultaneously closed the solenoid valves just upstream and downstream of the pressurized bed and monitored the collapse of the bed via the density output signal from the nuclear density gauge. As shown in Figure 3, as the first bubbles passed up through the bed, the initial bed collapse was rapid. After the bubbles had disappeared, the bed height fell more slowly as the dense phase gas permeated through the interstices of the particles.

Back extrapolation of this gradual collapse to time zero (measured from the time the bed was enclosed) yielded the bed height, L_D, which corresponded to the dense phase. Dense phase voidage

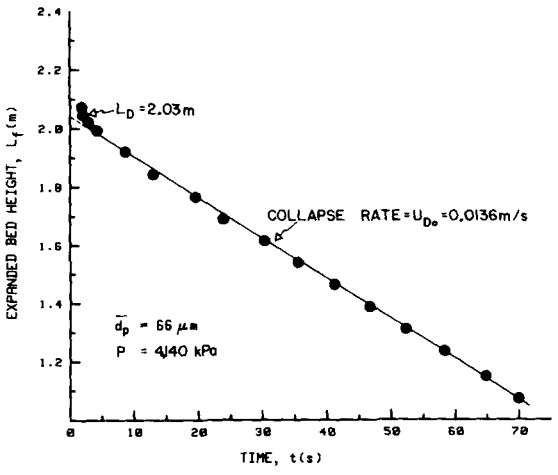

Figure 3. Collapsed bed experiment for determining dense phase properties.

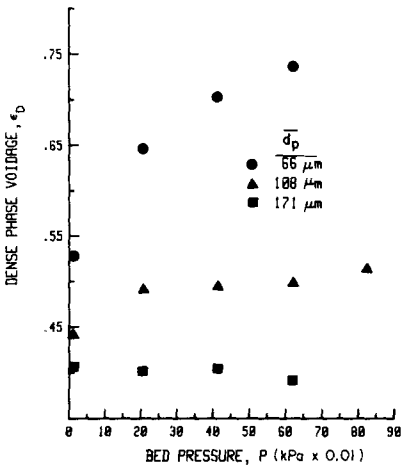

Figure 4. Effect of pressure and particle size on dense phase voidage.

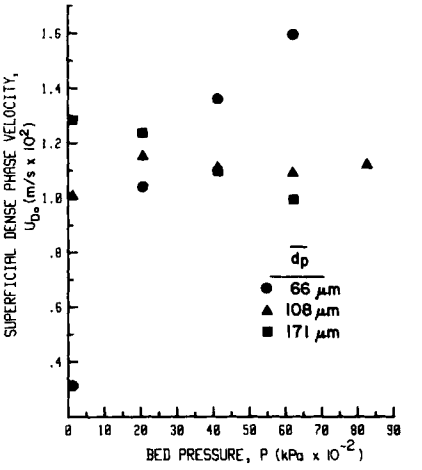

Figure 5. Effect of pressure and particle size on superficial dense phase gas velocity.

was determined as

$$\epsilon_D = 1 - \frac{W_T}{\rho_p L_D A_T} \tag{1}$$

The plot of collapsing bed height as a function of time was made from dynamic density measurements of the bed in which the density gauge was moved to various axial locations and the bed enclosures repeated from identical steady state flow conditions. The top of the bed was determined from the density gauge as that axial height where the measured bed density was one-half the bulk bed density. This experiment was done with the density gauge located at 0.02 m intervals from the expanded bed height before bed enclosure to the collapsed bed height after deaeration. This experiment was repeated at various high initial values of u_o for all pressures and powders. The dense phase voidage was found to be independent of u_o before bed enclosure.

The variation of ϵ_D with increasing pressures is shown in Figure 4. Clearly, the magnitude of the pressure effect on ϵ_D was strongly dependent on the mean particle size. There was a substantial pressure effect on ϵ_D for the 66 μm powder, a modest pressure effect for the 108 μm powder, and essentially no pressure effect for the 171 μm powder. The dense phase voidage of the 66 μm powder increased from 0.53 at 100 kPa pressure to 0.74 at 6,200 kPa. For the 108 μm powder, ϵ_D increased from 0.44 at 100 kPa pressure to 0.51 at 8,300 kPa. Measured ϵ_D for the 171 μm powder were found to be equivalent to ϵ_{mf} at all pressures.

The superficial dense phase gas velocity, u_{Do}, was taken as the rate of collapse of the dense phase following bed enclosure (Bohle and van Swaay, 1978; Drinkenberg and Rietema, 1973). Distributor design was such that plenum volume was negligible, thus eliminating the need to correct the measured collapse rate (Abrahamsen and Geldart, 1980; Dry et al., 1983).

The variation of u_{Do} with pressure is shown in Figure 5. Again, the effect of pressure was greater the smaller the mean particle size, having substantial effect on u_{Do} through the 66 μm powder and little effect on u_{Do} through the 108 μm powder. For the 66 μm powder, u_{Do} increased more than fivefold from 0.00313 m/s at 100 kPa to 0.0159 m/s at 6,200 kPa. For the 108 μm powder, u_{Do} increased only 11% at 8,300 kPa from its 0.010 m/s value at 100 kPa. For the 171 μm powder, $u_{Do} \approx u_{mf}$ and decreased slightly with increased pressure.

Bubble Absolute Rise Velocity, u_B, and Inferred Diameter, d_b

The mean absolute rise velocity of bubbles, u_B, near the top of the bed was determined from cross-correlation analysis of rapidly fluctuating differential pressure, ΔP, measurements (Fan et al., 1983; Sitnai et al., 1981; Sitnai, 1982; Swinehart, 1966). The mean bubble diameter, d_b, was inferred from u_B and known bed operating parameters. With the distance L between transducers DP3 and DP4 fixed, u_B is known if the transit time, τ, required to travel this distance is determined. Statistical cross-correlation techniques for determining τ are well established and described in the references cited above.

In the absence of wall effects and bubble-bubble interactions, bubble rise velocity is directly related to size. With $u_0 - u_{Do}$ known, and u_B measured by statistical techniques, d_b is inferred from a modified Davidson and Harrison (1963) bubble velocity equation:

$$d_b = \left[\frac{u_B - (u_o - u_{Do})}{0.711}\right]^2 \Big/ g \tag{2}$$

Although Eq. 2 is not correct from a continuity point of view (Turner, 1966), it has been shown to approximate the bubble ve-

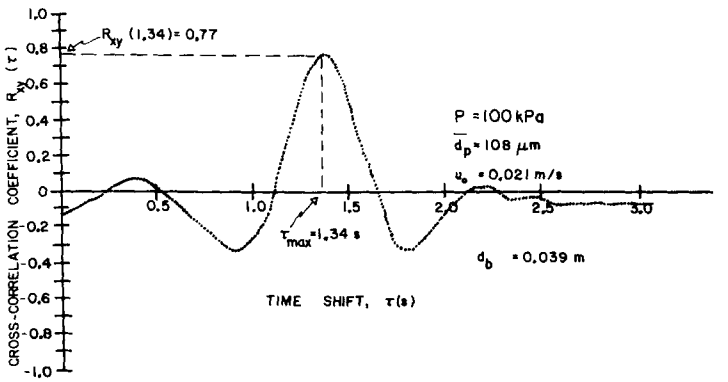

Figure 6. Statistical determination of mean bubble transit time near top of fluidized bed (z/d_T = 10.7).

locity/size relationship on a mean basis (Grace and Harrison, 1969). Strong correlations between ΔP signals (generally $R_{xy}(\tau) > 0.7$) indicated a very uniform u_B distribution, thus supporting the application of Eq. 2. When u_B is determined near the top of the bed (as is the case here), d_b inferred from the measurement is the largest size of bubbles in the bed. However, with increasing bubble size, there is a gradual transition to slug flow. The midpoint of this transition is at approximately $d_b/d_T = 0.36$ (Allahwala and Potter, 1979). For slug flow the absolute rise velocity is a function of bed diameter, d_T, rather than of bubble size. For the experimental reactor used in this study, $d_T = 0.128$ m and the midpoint of the transition to slug flow would be expected to correspond to a bubble size of about 0.046 m. Caution should be exercised in interpreting inferred bubble sizes that are larger than this value.

The cross-correlation function for the 108 μm powder at atmospheric pressure is illustrated in Figure 6 for the case of u_o = 0.021 m/s. The transit time, τ = 1.34 s, was given by the maximum of the cross-correlation coefficient, R_{xy} (1.34) = 0.77, which indicates a strong correlation between ΔP measurements. The corresponding mean bubble size near the top of the bed was inferred to be d_b = 0.039 m.

The cross-correlation technique provided an excellent method for determining u_B for bubbles larger than approximately 0.01 m. For bubbles smaller than this size, the correlation between ΔP measurements was too weak to calculate u_B accurately. Measured u_B fell in the range 0.20–0.55 m/s.

Bubble sizes near the top of the bed were determined for the 66, 108 and 171 μm powders at pressure up to 8,300 kPa. The bubble diameters are illustrated as a function of excess superficial gas velocities, $u_o - u_{mf}$ (Figure 7). Figure 7 illustrates the effect of pressure for a given particle size. As illustrated in Figure 7a, increased pressure had a significant influence on bubble size for the 66 μm powder. Bubble size was reduced 80% from approximately 0.05 m at 100 kPa pressure to approximately 0.01 m at 2,100 kPa for equivalent $u_o - u_{mf}$. Bubbles smaller than 0.01 m resulted in operating conditions above 2,100 kPa pressure for all u_o and for $u_o - u_{mf} < 0.03$ m/s at 2,100 kPa. The effect of pressure on bubble size was not as extreme for the 108 μm powder (Figure 7b) as bubble size decreased approximately 12% from 0.05 m at 100 kPa pressure to approximately 0.042 m at 2,100 kPa. The overall percent reduction in bubble size from that at 100 kPa pressure was approximately 27% to 0.035 m at 4,100 kPa, 50% to 0.024 m at 6,200 kPa, and 79% to bubbles less than 0.01 m dia. at 8,300 kPa. As illustrated in Figure 7c, the pressure effect on the 171 μm powder was minimal. At $u_o - u_{mf}$ = 0.015 m/s, d_b was reduced approximately 27% from 100 kPa to 6,200 kPa operation. This corresponded to the approximate 50% reduction in size for bubbles in the 108 μm powder at equivalent conditions.

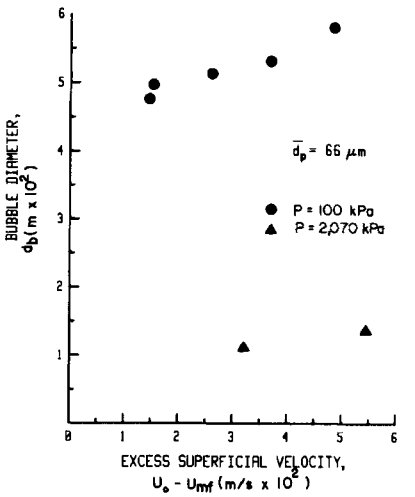

Figure 7a. Effect of gas flow and pressure on inferred bubble size, 66 μm powder, z/d_T = 10.7.

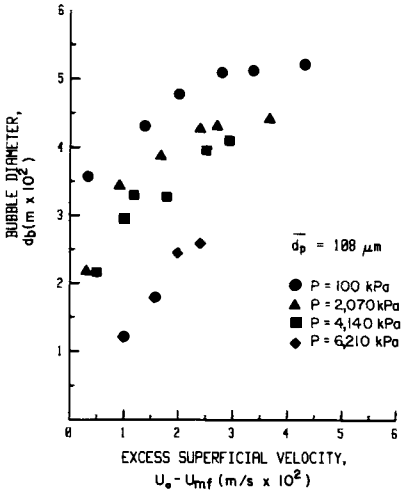

Figure 7b. Effect of gas flow and pressure on inferred bubble size, 108 μm powder, z/d_T = 10.7.

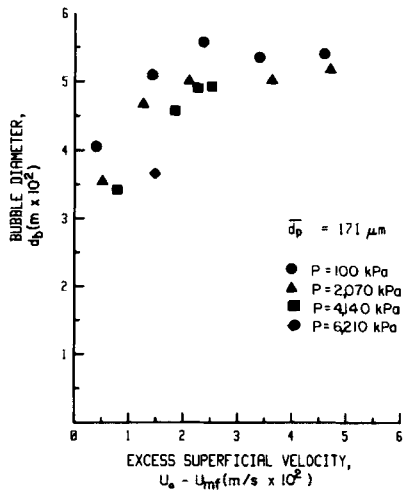

Figure 7c. Effect of gas flow and pressure on inferred bubble size, 171 μm powder, z/d_T = 10.7.

DISCUSSION

On Predicting Dense Phase Voidage, ϵ_D, and Superficial Dense Phase Gas Velocity, u_{Do}

Abrahamsen and Geldart (1980b) have presented a correlation for predicting ϵ_D in fluidized beds of fine powders:

$$\frac{1-\epsilon_{mf}}{1-\epsilon_D} = 2.54\,\frac{\rho_g^{0.016}\mu^{0.066}\exp(0.090\,F)}{d_p^{0.1}g^{0.118}(\rho_p-\rho_g)^{0.118}L_{mf}^{0.043}} \qquad (3)$$

Although the accuracy of this equation has not been evaluated at pressure, density effects are included and such an evaluation can be done. With experimental values for ϵ_{mf} and L_{mf} the predicted values of ϵ_D are compared in Figure 8 to those determined experimentally for the 66 and 108 μm powders at all pressures. Although predicted and experimental values for the 108 μm powder are in good agreement, Eq. 3 largely underestimates the effect of gas density on ϵ_D for the 66 μm powder.

With ϵ_{mf}, u_{mf}, and ϵ_D known, Abrahamsen and Geldart (1980b) proposed that u_{Do} could be determined from:

$$\left(\frac{\epsilon_D}{\epsilon_{mf}}\right)^3\frac{1-\epsilon_{mf}}{1-\epsilon_D} = \left(\frac{u_{Do}}{u_{mf}}\right)^{0.70} \qquad (4)$$

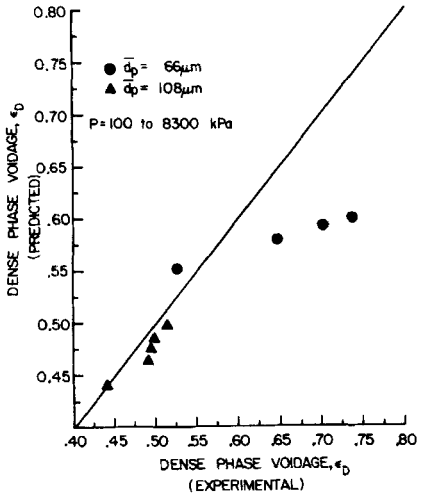

Figure 8. Comparison of experimental values for dense phase voidage with those predicted by Abrahamsen and Geldart (1980b), Eq. 3.

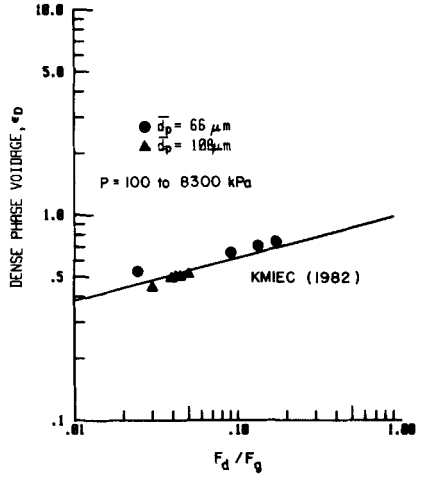

Figure 10. Comparison of experimental values for dense phase voidage with those predicted by Kmiec (1982), Eq. 6.

Experimental measurements are compared to predicted values in Figure 9. Predicted and actual values are in reasonable agreement for both powders at all pressures.

An alternative relationship has recently been proposed by Kmiec (1982):

$$\epsilon_D = (18\,Re_p + 2.7\,Re_p^{1.687})^{0.209}/AR^{0.209} \tag{5}$$

obtained from:

$$\epsilon_D = \left(\frac{F_D}{F_G}\right)^{0.209} \tag{6}$$

where

$$\frac{F_D}{F_G} = \frac{3}{4} C_D \frac{Re_p^2}{AR} \tag{7}$$

Equation 6 is shown in Figure 10 along with the experimentally determined values for ϵ_D and F_D/F_G. Predicted and experimental results are in excellent agreement with Eq. 6 and 7, adequately predicting the $\epsilon_D - u_{Do}$ relationship at all pressures for both the 66 and 108 μm powders.

On Reduced Bubble Flow Limiting Bubble Growth

A theory to explain smaller bubbles in high pressure systems is that smaller bubbles result from smaller bubble flow rates (i.e., smaller u_{Bo}). It has been shown that the fluidization of fine powders resulted in $u_{Do} > u_{mf}$ and that smaller particle sizes and higher operating pressures increased this effect. Superficial dense phase gas velocities as high as $u_{Do} = 25\,u_{mf}$ have been reported (Rowe et al., 1978). In this work (Figure 5) $u_{Do} > 7\,u_{mf}$ for the 66 μm powder.

To test the validity of this theory it has been applied to estimate mean bubble diameters near the top of the bed. It is assumed that $u_{Bo} = u_o - u_{Do}$ and that total coalescence of the bubbles has occurred, resulting in a single train of bubbles rising along the center line of the bed. The theory follows that given by Mori and Wen (1975) with the exception that in general $u_{Do} \neq u_{mf}$. The expected bubble diameter at the measuring height is given by

$$d_b = d_{bm} - (d_{bm} - d_{bo}) \exp(-0.3z/d_T) \tag{8}$$

where

$$d_{bm} = 1.638(A_T(u_o - u_{Do}))^{0.4} \tag{9}$$

$$d_{bo} = 0.376(u_o - u_{Do})^2 \tag{10}$$

$z/d_T = 10.7$, for the measurement height here.

Predicted and measured bubble sizes are shown in Figure 11 for all powders at all pressures. In general, the predicted bubble diameters are larger than the measured values. Predicted sizes for the 66 μm powder at 2100 kPa were 8 times the observed values. It is clear the smaller bubbles observed at higher pressure resulted from factors other than reduced bubble flow, u_{Bo}. At a given pressure, d_b did not increase above a given u_{Bo} and for a given u_{Bo}, bubbles were smaller at higher pressures. These findings indicated that a maximum limiting bubble size decreased with increased pressure.

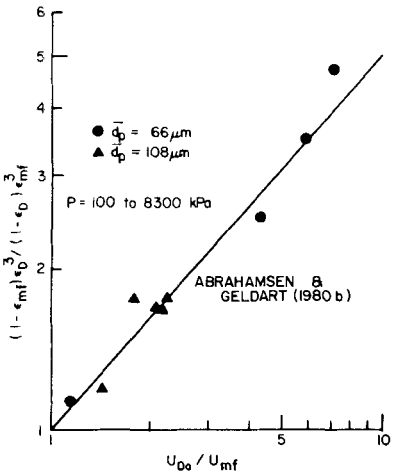

Figure 9. Experimental voidage-velocity results compared with those predicted by Abrahamsen and Geldart (1980b), Eq. 4.

On Particle "Pickup" Limiting Bubble Growth

Harrison et al. (1961) postulated that a stable limiting bubble size exists for fluidized beds. When too large a bubble is formed, it breaks up into smaller bubbles with breakup occurring by solid particles being "picked up" into the bubble, thereby splitting the bubble from below. The stable limiting size is dependent on particle size, particle density, gas density, and gas viscosity. According to the theory, as a bubble rises through a bed of particles, an internal circulation pattern is set up within the bubble with a circulation velocity that is approximately equal to the bubble rise velocity:

$$u_c \approx u_{br} = 0.711(gd_b)^{0.5} \tag{11}$$

If the circulation velocity exceeds the terminal velocity of the particles u_t, the particles can no longer fall out of the bubble. The particles are then drawn into the bubble from below, thereby causing bubble breakup. The conditions for bubble stability are

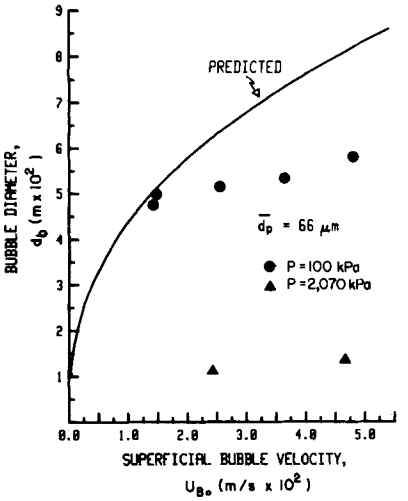

Figure 11a. Effect of superficial bubble velocity and pressure on inferred bubble size near top of bed, 66 μm powder, z/d_T = 10.7.

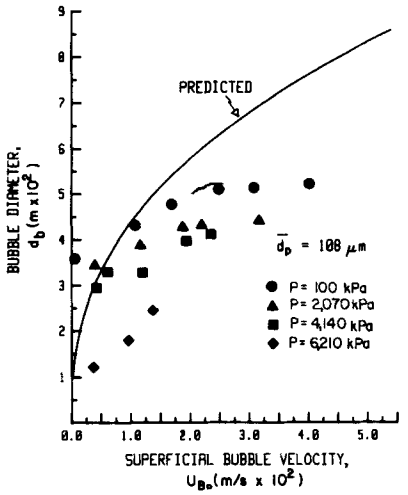

Figure 11b. Effect of superficial bubble velocity and pressure on inferred bubble size near top of bed, 108 μm powder, z/d_T = 10.7.

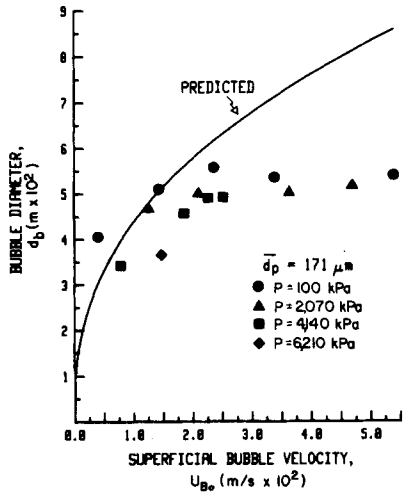

Figure 11c. Effect of superficial buble velocity and pressure on inferred bubble size near top of bed, 171 μm powder, z/d_T = 10.7.

$$u_c < u_t \text{ stable bubble}$$

$$u_c = u_t \text{ maximum size of stable bubble}$$

$$u_c > u_t \text{ unstable bubble}$$

Support for this proposed mechanism of bubbles splitting from below was obtained from observed bubble splitting in bubbling fluidized lead shot/water systems (Harrison et al., 1961). Bubbles were seen to divide when particles from the wake were carried upward into bubble by the fluidizing fluid. Recently, this phenomenon has been observed in pressurized Group A gas/solid systems (Barreto et al., 1983).

Fluidization studies with fine powders in a pilot gas-solid fluidized bed (Matsen, 1973) indicated that the directional relationship between particle size and maximum stable bubble size as predicted by this theory was correct. However, a comparison of the experimental stable bubble size observations with those theoretically predicted showed that for a given system, the Harrison et al. (1961) theory consistently under predicted the bubble size.

For the fluidization systems in this work, the maximum stable bubble diameter, according to the particle "pickup" theory, is given by

$$d_{b_{max}} = 0.135 \left[\frac{(\rho_p - \rho_g)^2}{\rho_g \mu}\right]^{2/3} d_{ps}^2 g^{1/3} \qquad (12)$$

$$0.4 < Re_p < 500$$

A comparison of the theoretically predicted and experimentally determined maximum bubble sizes for the 66 μm, 108 μm, and 171 μm powders at pressures up to 6200 kPa is given in Figure 12. The mean bubble size measurements were made near the top of the fluidized bed at the highest gas velocities possible. Clearly, the directional trend of smaller stable bubble diameters with smaller particles is adequately predicted; however, quantitative predictions of maximum stable bubble size are underestimated to a large degree. Experimentally inferred bubble sizes for the 66 μm powder were fourfold larger than predicted values. Furthermore, the increased density effect (increased pressure effect) on maximum bubble size obtained for the 108 and 171 μm powders was overestimated.

Quantitatively, the particle "pickup" theory does not adequately predict the maximum stable bubble size for a given system. However, the qualitative directional effect of particle size and gas density on maximum stable bubble size is correct.

On Taylor Instabilities Limiting Bubble Growth

Another explanation for limited bubble growth in high pressure fluidized beds is given in terms of an increased instability of the

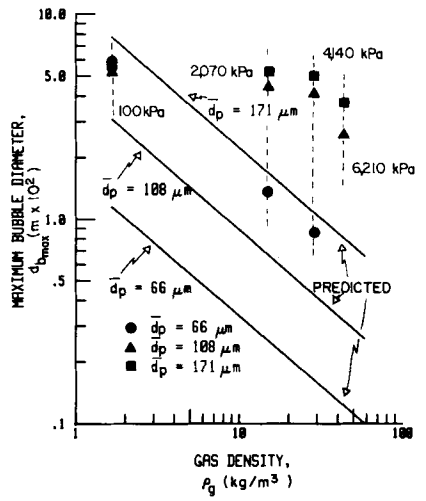

Figure 12. Comparison of inferred maximum bubble sizes with those predicted by the particle "pickup" theory of Harrison et al. (1961).

roof of the bubbles at higher pressures. Photographs (Rowe and Partridge, 1964) of air bubbles in water and X-rays (Rowe and Partridge, 1965) of gas bubbles in fluidized beds have shown that an initial disturbance, indenting the upper surface of a bubble, grows to split it from above. It has been suggested that this splitting results from a Taylor instability i.e., from an instability of the type discussed by Taylor (Taylor, 1950) wherein a heavy fluid overlies a lighter one (Clift and Grace, 1972).

In applying the Taylor theory to bubbles in fluidized beds, the dense phase over the bubble is the heavy fluid and the gas in the bubble is the lighter fluid. An initial, random, small disturbance with amplitude η_o is assumed to perturb the essentially horizontal upper surface of the bubble, and to grow according to

$$\eta = \eta_o e^{nt} \tag{13}$$

where n is a parameter depending on the physical properties of the system and on the wavelength of the disturbance. In gas-fluidized beds, n may be obtained from (Clift et al., 1974)

$$R^4 + 2R^2 - 4R + 1 - \frac{g}{k^3 \nu_p^2} = 0 \tag{14}$$

where

$$k = 2\pi/\lambda \tag{15}$$

is the wave number of the disturbance and

$$R = 1 + n(\nu_p k^2) \tag{16}$$

A numerical solution for Eq. 14 has been presented elsewhere (Clift et al., 1974). For each ν_p, there is a specific wavelength called the "most dangerous" wavelength, λ_{max}, which exhibits the maximum growth rate. Guedes de Carvalho (1981) estimated ν_p of a high pressure fluidized-bed system from ϵ_D measurements and concluded that the small increase in growth rate of the most dangerous disturbance, n_{max}, with increased pressure, could not account for the marked change in bubbling behavior of the high pressure fluidized bed. The highest pressure investigated was 2,000 kPa, and ϵ_D increased only 15% over this pressure range.

Here we consider a more detailed interpretation of the Taylor instability theory as applied to a fluidized bed operating at pressures up to 8,300 kPa with ϵ_D changes of 40% (Figure 4). Clift et al. (1974) have noted that disturbances initiated on the roof of a bubble are swept around the periphery. In practice, a bubble does not split unless the disturbance has grown sufficiently before the tip of the growing spike reaches the side of the bubble. The likelihood of splitting can be estimated by comparing the time constant for the growth of a disturbance, i.e., $t_e = 1/n$, with the maximum time available for growth (Clift et al., 1974)

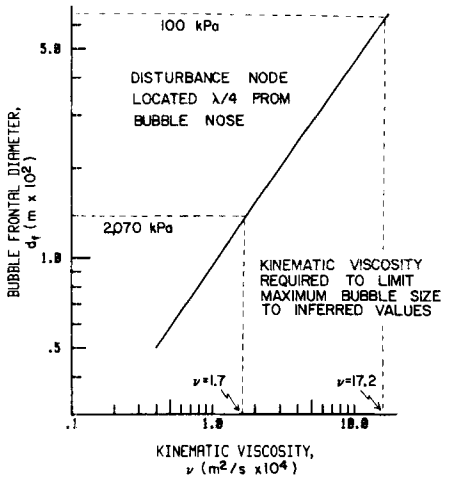

Figure 13. Maximum stable bubble frontal diameter that, according to the Taylor instability theory, exists in a bed with the corresponding kinematic viscosities.

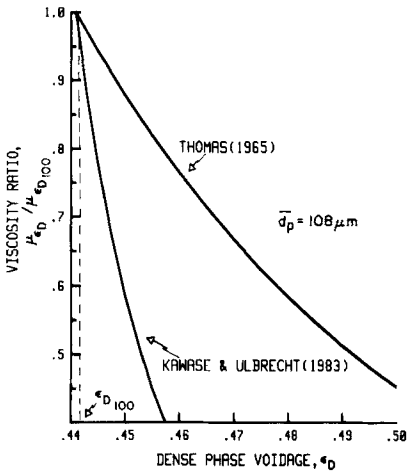

Figure 14. Predicted dynamic viscosity of the dense phase relative to that at 100 kPa pressure. Viscosity is a function of voidage, 108 μm powder.

$$t_{am} = \sqrt{\frac{R_o}{g}} \ln\left[\frac{1}{\tan(\theta/2)}\right] \tag{17}$$

If the time available for growth, t_{am}, is greater than the required growth time, t_e, the bubble is liable to split. Otherwise, the disturbance grows so slowly that it does not achieve an amplitude large enough to cause splitting, before it is swept around to the bubble equator.

Observations of splitting bubbles suggest that disturbances usually develop in a regular pattern to either side of the node. Assuming a node is located $\lambda/4$ from the bubble nose so that the node is an antinode in the initial disturbance

$$\theta = \frac{\lambda}{2R_o} = \frac{\pi}{kR_0} \tag{18}$$

The relationship between stable bubble frontal diameter, d_f, and the corresponding ν_p is shown in Figure 13. It is clear from Eq. 14 and from Figure 13 that ν_p is the dominant factor determining the growth of the instabilities, the most dangerous wavelength, λ_{max}, and hence the maximum stable bubble size $d_{b_{max}}$. Thus, prediction of the effect of system properties on bubble stability depends on prediction of the effect of system properties on ν_p. Smaller particles and higher pressures are known to lower ν_p (King et al., 1981) and thus, according to the Taylor instability theory, result in a smaller d_f. The major difficulty in testing the ability of using the Taylor instability theory to explain observed limited bubble growth in high pressure fluidized beds is the lack of data, correlations, and theory

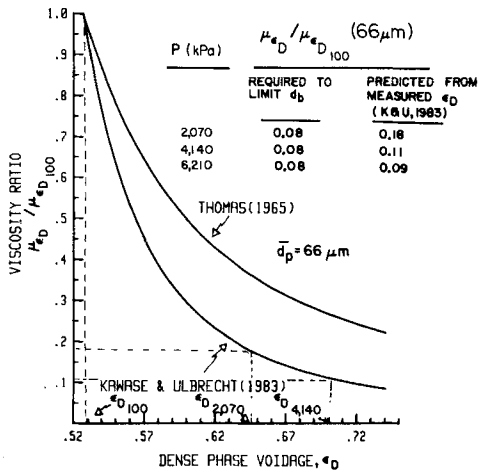

Figure 15. Predicted dynamic viscosity of the dense phase relative to that at 100 kPa pressure. Viscosity is a function of voidage, 66 μm powder.

TABLE 3. PARAMETERS FOR TAYLOR INSTABILITY PHENOMENON

	Measured		Required		
P, kPa	ϵ_D	d_f, m	ν_p m²/s · 10^4	μ_D, Pa · s	$\mu_D/\mu_{D_{100\ kPa}}$
66 μm Powder					
100	0.527	0.064	17.2	0.692	1.00
2,100	0.646	0.014	1.7	0.052	0.08
4,100	0.702	<0.01	1.1	0.028	<0.04
6,200	0.76	<0.01	1.1	0.028	<0.04
108 μm Powder					
100	0.441	0.057	14.6	0.693	1.00
2,100	0.491	0.049	11.4	0.494	0.71
4,100	0.494	0.045	10.2	0.437	0.63

relating the kinematic viscosity of the dense phase, ν_p, to system parameters. The relationship between stable bubble frontal diameters, d_f, and the corresponding ν_p is shown in Figure 13. If the observed decrease in d_b with pressure results from Taylor instabilities, we can find from Figure 13 the effective ν_p corresponding to the observed $d_{b_{max}}$ for each d_p and pressure. In a qualitative sense, the variation with pressure of the deduced ν_p must be consistent with the observed variation in ϵ_D.

Actual bubble volume is approximately 75% of that of a sphere of diameter equivalent to d_f (Rowe and Partridge, 1966). Hence d_f and d_b are related:

$$d_f = 1.1\, d_b \tag{19}$$

For the 66 μm powder (Figure 7a) $d_{b_{max}}$ observed was 0.058 m at 100 kPa and 0.014 m at 2100 kPa. Likewise, for the 108 μm powder (Figure 7b), the observed $d_{b_{max}}$ decreased from 0.052 m at 100 kPa to 0.041 m at 4100 kPa.

Viscosity measurements in gas-fluidized beds have been limited to pressures below 2,100 kPa (Grace, 1970; King et al., 1981; Schugerl, 1971) and $\epsilon_D < 0.55$. Furthermore, no correlations of the data are available. Estimation of ϵ_D is limited either to reported experimental measurements or correlations developed for liquid-solid systems.

The required viscosities ($\mu_D = 0.4$ to 0.7) for the 108 μm powder at all pressures and the 66 μm powder at 100 kPa are within the range of experimentally determined viscosities for gas-fluidized beds. However, the viscosity ($\mu_D = 0.052$) required to limit bubble size for the 66 μm powder is lower than any previously measured values.

The equations of Thomas (1965) and Kawase and Ulbrecht (1983) for predicting relative viscosity ($\mu_{\epsilon_D}/\mu_{\epsilon_{D100\ kPa}}$) are shown in Figures 14 and 15 as a function of ϵ_D for the 108 and 66 μm powders. For the 108 μm powder (Figure 14) at 4,100 kPa, the equation of Thomas (1965) predicts μ_D to be 48% of μ_D at 100 kPa, $\mu_{D_{100\ kPa}}$. Kawase and Ulbrecht's equation predicts even lower viscosities. These decreases in viscosity are more than the decrease required (required $\mu_D/\mu_{D_{100\ kPa}} = 63\%$) to limit bubble size to $d_f = 0.045$ m (see Table 3). For the 66 μm powder (Figure 15), Thomas' equation estimates μ_D to be 36, 26 and 23% of $\mu_{D_{100\ kPa}}$ at 2,100, 4,100 and 6,200 kPa, respectively, while the corresponding percentages from Kawase and Ulbrecht's equation are 18, 11 and 9%. From Table 3, it is clear that $\mu_{D_{2,100\ kPa}}/\mu_{D_{100\ kPa}} = 0.08$ is required to satisfy the Taylor instability criterion and ensure bubbles no larger than $d_f = 0.014$ m at 2,100 kPa. This required viscosity change occurred at 6,200 kPa bed operation (according to Kawase and Ulbrecht's equation). However, the bed viscosity at 2,100 kPa was estimated to be twofold larger than this.

These results indicate that the observed variation in ϵ_D with pressure is consistent with that required to limit bubble size according to the Taylor instability theory. However, caution must be exercised on three accounts: (1) as discussed previously, the $d_{b_{max}}$ observed at 100 kPa may have been limited by the bed diameter. If this was the case, bubbles at 100 kPa would be larger than those measured and, hence, the required change in ϵ_D would be larger also. (2) The bed voidage just above rising bubbles may be greater than ϵ_D. Such a voidage increase above rising bubbles is predicted according to Jackson's (1963) analysis of bubble motion. This phenomenon would increase the effect of Taylor instabilities in limiting bubble growth in gas-solid fluidized beds. (3) The correlations used to estimate variations in μ_D from variations in ϵ_D were developed for systems of solid spheres in liquids and are probably not directly applicable to gas-solid systems of granular powders. A relationship

$$\frac{\mu_D}{\mu_g} = f(d_p, \rho_p, \phi, \epsilon) \tag{20}$$

for gas-solid systems is needed before $d_{b_{max}}$ can be more accurately estimated. Future research should define Eq. 20.

NOTATION

A_T = cross-sectional area of the bed
AR = $g\, d_p{}^3(\rho_p - \rho_g)\rho_g/\mu_g{}^2$, Archimedes number
C_D = $24(1 + 0.15\, Re_p^{0.687})/Re_p$; $Re_p \leq 500$, drag coefficient for an isolated free particle at flow
d_b = bubble diameter (volume equivalent sphere)
d_{bo} = initial bubble diameter
d_{bm} = maximum bubble diameter expected from Eq. 9
$d_{b_{max}}$ = maximum stable bubble diameter, independent of gas superficial velocity
d_f = frontal bubble diameter
d_p = particle diameter
d_T = fluidized-bed diameter (inside)
F_d = particle apparent drag force
F_g = particle effective gravitational force
g = gravitational constant
k = wave number of a disturbance
L = distance between high frequency transducers DP3 and DP4
L_d = dense phase bed height
L_f = expanded bed height
L_{mf} = minimum fluidization bed height
l = distance spanned by DP3 and DP4
M_w = molecular weight
P = pressure
R = defined in Eq. 16
R_o = frontal radius of a bubble
R_{xy} = discretized cross-correlation coefficient
Re_p = $u_{Do}\, d_p\, \rho_g/\mu_g$, particle Reynolds number
t = time
t_{am} = maximum time available for disturbance growth
t_e = time for disturbance to grow by a factor e
u_o = superficial gas velocity
u_B = absolute bubble rise velocity
u_{Bo} = superficial bubble phase velocity
u_{Br} = bubble natural rising velocity
u_c = circulation velocity
u_{Do} = superficial dense phase velocity
u_{mf} = minimum fluidization velocity
u_t = particle terminal velocity
W_T = total solids weight in bed
z = axial distance

Greek Letters

ΔP = differential pressure
ϵ_D = dense phase voidage
ϵ_{mb} = bed voidage at minimum bubbling
ϵ_{mf} = bed voidage at minimum fluidization
η = amplitude of disturbance
η_o = initial amplitude of disturbance
θ = angular position where disturbance originates
λ = wavelength of disturbance
λ_{max} = most dangerous wavelength
μ = gas viscosity
μ_D = dense phase viscosity
$\mu_{D\,100\text{ kPa}}$ = dense phase viscosity at 100 kPa pressure
$\mu_{\epsilon D}$ = viscosity corresponding to ϵ_D
$\mu_{\epsilon D\,100\text{ kPa}}$ = viscosity corresponding to ϵ_D at 100 kPa pressure
ν_p = kinematic dense phase viscosity
ρ_g = gas density
ρ_p = particle density
τ = time lag for cross-correlation analysis
ϕ = particle sphericity

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